Q. 10 Distinguish between :

(1) mass and weight (2) universal gravitational constant and gravitational acceleration of the earth.

Ans.

(1) Mass	Weight
1. The mass of a body is the amount of matter present in it.	1. The weight of a body is the force with which the earth attracts it.
2. It has magnitude, but not direction.	2. It has both magnitude and direction.
3. It does not change from place to place.	3. It changes from place to place.
4. It can never be zero.	4. It is zero at the centre of the earth.
5. Its SI unit is the kilogram.	5. Its SI unit is the newton.

(2) Universal gravitational constant	Gravitational acceleration of the earth
1. The universal gravitational constant numerically equals the force of attraction between two unit masses separated by a unit distance.	1. The gravitational acceleration of the earth is the acceleration produced in a body due to the gravitational force of the earth.

2. Its value remains constant throughout the universe.	2. Its value changes from place to place.
3. It has magnitude but not direction.	3. It has both magnitude and direction.
4. Its SI unit is N·m²/kg².	4. Its SI unit is m/s².

Q. 11 Solve the following examples :

$(G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2,\ g = 9.8\ m/s^2)$

(1) The time taken by the earth to complete one revolution around the Sun is 3.156×10^7 s. The distance between the earth and the Sun is 1.5×10^{11} m. Find the speed of revolution of the earth.

Solution : Data : $T = 3.156 \times 10^7$ s, $r = 1.5 \times 10^{11}$ m, $v = ?$

$$v = \frac{2\pi r}{T} = \frac{2 \times 3.142 \times 1.5 \times 10^{11}\ m}{3.156 \times 10^7\ s}$$

$= 2.987 \times 10^4$ **m/s = 29.87 km/s**

This is the speed of revolution of the earth.

(2) Assuming that the earth performs uniform circular motion around the Sun, find the centripetal acceleration of the earth. [Speed of the earth = 3×10^4 m/s, distance between the earth and the Sun = 1.5×10^{11} m]

Solution : Data : $v = 3 \times 10^4$ m/s, $r = 1.5 \times 10^{11}$ m

Centripetal force $= \frac{mv^2}{r} = ma$

∴ Centripetal acceleration of the earth,

$$a = \frac{v^2}{r} = \frac{(3 \times 10^4\ m/s)^2}{1.5 \times 10^{11}\ m} = \frac{3 \times 3}{1.5} \times 10^{-3}\ m/s^2$$

$= 6 \times 10^{-3}$ **m/s²**

It is directed towards the centre of the Sun.

Use of ICT : **(Textbook page 1)**

Collect videos and ppts about the gravitational force of different planets.

(3) What will be the gravitational force on 60 kg man on the Moon, Mars and Jupiter? Are they the same? Why?

M (Moon) = 7.36×10^{22} kg, R (Moon) = 1.74×10^6 m, M (Mars) = 6.4×10^{23} kg, R (Mars) = 3.395×10^6 m, M (Jupiter) = 1.9×10^{27} kg. R (Jupiter) = 7.15×10^7 m, $G = 6.67 \times 10^{-11}$ N·m²/kg²

Solution :

(1) Data : $m_1 = 60$ kg, $m_2 = 7.36 \times 10^{22}$ kg, $R = 1.74 \times 10^6$ m

$$\therefore F = \frac{Gm_1m_2}{R^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 60\ kg \times 7.36 \times 10^{22}\ kg}{(1.74 \times 10^6\ m)^2}$$

$= 97.29$ N

On the moon's surface, the gravitational force on the man due to the moon = 97.29 N.

(2) Data : $m_1 = 60$ kg, $m_2 = 6.4 \times 10^{23}$ kg, $R = 3.395 \times 10^6$ m

$$\therefore F = \frac{Gm_1m_2}{R^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 60\ kg \times 6.4 \times 10^{23}\ kg}{(3.395 \times 10^6\ m)^2}$$

$= 222.2$ N

On the surface of Mars, the gravitational force on the man due to Mars = 222.2 N.

(3) Data : $m_1 = 60$ kg, $m_2 = 1.9 \times 10^{27}$ kg, $R = 7.15 \times 10^7$ m

$$\therefore F = \frac{Gm_1m_2}{R^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 60\ kg \times 1.9 \times 10^{27}\ kg}{(7.15 \times 10^7\ m)^2}$$

$= 1487$ N

On the surface of Jupiter, the gravitational force on the man due to Jupiter = 1487 N.

Thus, the forces on the man are not the same because the ratio (M/R^2) is not the same in the case of the moon, Mars and Jupiter.

***(4) The masses of the earth and moon are 6×10^{24} kg and 7.4×10^{22} kg, respectively. The distance between them is 3.84×10^5 km. Calculate the gravitational force of attraction between the two. Use $G = 6.7 \times 10^{-11}$ N·m² kg⁻²**

Solution : Data : $m_1 = 6 \times 10^{24}$ kg, $m_2 = 7.4 \times 10^{22}$ kg, $r = 3.84 \times 10^5$ km $= 3.84 \times 10^8$ m, $G = 6.7 \times 10^{-11}\ N \cdot m^2\ kg^{-2}$, $F = ?$

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.7 \times 10^{-11}\ N \cdot m^2\ kg^{-2} \times 6 \times 10^{24}\ kg \times 7.4 \times 10^{22}\ kg}{(3.84 \times 10^8\ m)^2}$$

$$= \frac{6.7 \times 6 \times 7.4 \times 10^{35}}{3.84 \times 3.84 \times 10^{16}} N = \mathbf{2.017 \times 10^{20}\ N}$$

This is (the magnitude of) the gravitational force between the earth and the moon.

(5) Spheres A and B of uniform density have masses 1 kg and 100 kg respectively. Their centres are separated by 100 m. (i) Find the gravitational force between them. (ii) Find the gravitational force on A due to the earth. (iii) Suppose A and B are initially at rest and A can move freely towards B. What will be the velocity of A one second after it starts moving towards B? How will this velocity change with time? How much time will A take to move towards B by 1 cm? (iv) If A begins to fall, starting from rest, due to the earth's downward pull, what will be its velocity after one second? How much time will it take to fall through 1 cm?

[M(earth) $= 6 \times 10^{24}$ kg, R(earth) $= 6400$ km]

Solution : Data : $m_1 = 1$ kg, $m_2 = 100$ kg, $r = 100$ m, $M = 6 \times 10^{24}$ kg, $R = 6400$ km $= 6400 \times 10^3$ m, $t = 1$ s, $s = 1$ cm $= 1 \times 10^{-2}$ m, $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$

$F_1 = ?, F_2 = ?, v_1 = ?, t_1 = ?, v_2 = ?, t_2 = ?$

(i) $$F_1 = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 1\ kg \times 100\ kg}{(100\ m)^2}$$

$$= \mathbf{6.67 \times 10^{-13}\ N.}$$

(ii) $$F_2 = \frac{Gm_1M}{R^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 1\ kg \times 6 \times 10^{24}\ kg}{(6400 \times 10^3\ m)^2}$$

$$= \frac{40.02 \times 10^{13}}{6.4 \times 6.4 \times 10^{12}} N = \mathbf{9.77\ N}$$

This is far greater than F_1.

(iii) Ignoring variation of acceleration with distance,

$$v_1 = u_1 + at = 0 + \frac{F_1}{m_1}t = \frac{6.67 \times 10^{-13}\ N}{1\ kg} \times 1\ s$$

$$= \mathbf{6.67 \times 10^{-13}\ m/s.}$$

This velocity is directed from A to B. As the separation between A and B decreases, the acceleration of A and hence the velocity of A will increase.

Ignoring variation of acceleration with distance,

$$s_1 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{F_1}{m_1}t_1^2$$

$$\therefore t_1^2 = \frac{2m_1s_1}{F_1}$$

$$\therefore t_1^2 = \frac{2 \times 1\ kg \times 10^{-2}\ m}{6.67 \times 10^{-13}\ N} = 3 \times 10^{10}\ s^2$$

$$\therefore t_1 = \mathbf{1.732 \times 10^5\ s.}$$

(iv) $$v_2 = u_2 + at = 0 + gt = \frac{F_2}{m_1}t$$

$$= \frac{9.77\ N}{1\ kg} \times 1\ s = \mathbf{9.77\ m/s\ (downward).}$$

$[|v_2| \gg |v_1|]$

$$s_2 = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}gt_2^2 = \frac{1}{2}\frac{F_2}{m_1}t_2^2$$

$$\therefore t_2^2 = \frac{2s_2m_1}{F_2} = \frac{2 \times 10^{-2}\ m \times 1\ kg}{9.77\ N} = 0.205\ s^2$$

$$\therefore t_2 \doteqdot \mathbf{0.453\ s} \quad [t_1 \gg t_2]$$

(6) Two spheres of uniform density have masses 10 kg and 40 kg. The distance between the centres of the spheres is 200 m. Find the gravitational force between them.

Solution : Data : $m_1 = 10$ kg, $m_2 = 40$ kg, $r = 200$ m, $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$, $F = ?$

$$F = \frac{Gm_1m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 10\ kg \times 40\ kg}{(200\ m)^2}$$

$$= \frac{6.67 \times 10^{-11} \times 4 \times 10^2\ N}{4 \times 10^4} = 6.67 \times 10^{-13}\ N$$

The gravitational force between the two spheres $= 6.67 \times 10^{-13}$ N.

(7) Find the gravitational force between a man of mass 50 kg and a car of mass 1500 kg separated by 10 m.

Solution : Data : $m_1 = 50$ kg, $m_2 = 1500$ kg, $r = 10$ m, $G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$, $F = ?$

$F = \frac{Gm_1m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 50 \text{ kg} \times 1500 \text{ kg}}{(10 \text{ m})^2}$

$= 5.0025 \times 10^{-8}$ N

The gravitational force between the man and the car = 5.0025×10^{-8} N.

- Use your brain power! **(Textbook page 6)**

Q. Assuming the acceleration in Example 2 above remains constant, how long will Mahendra take to move 1 cm towards Virat?

Ans. Here, $u = 0$

$\therefore s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2$

$\therefore t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 1 \times 10^{-2} \text{ m}}{5.34 \times 10^{-9} \text{ m/s}^2}}$

$= \sqrt{0.3745 \times 10^7 \text{ s}^2} = \sqrt{3.745 \times 10^6}$ s

$= 1935$ s $=$ **32 minutes 15 seconds.**

(8) Find the magnitude of the gravitational force between the sun and the earth. (Mass of the Sun = 2×10^{30} kg, mass of the earth = 6×10^{24} kg and the distance between the centres of the Sun and the earth = 1.5×10^{11} m, $G = 6.67 \times 10^{-11}$ N·m² / kg²)

Solution : Data : $m_1 = 2 \times 10^{30}$ kg, $m_2 = 6 \times 10^{24}$ kg, $r = 1.5 \times 10^{11}$ m, $G = 6.67 \times 10^{-11}$ N·m² / kg², $F = ?$

$F = G\frac{m_1m_2}{r^2}$

$= \frac{6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 2 \times 10^{30} \text{kg} \times 6 \times 10^{24} \text{kg}}{(1.5 \times 10^{11} \text{ m})^2}$

$= \frac{6.67 \times 2 \times 6}{1.5 \times 1.5} \times 10^{21} \text{ N} = 35.57 \times 10^{21}$ N

$\therefore F = 3.557 \times 10^{22}$ N

The magnitude of the gravitational force between the sun and the earth = 3.557×10^{22} N.

***(9) The mass of the earth is 6×10^{24} kg. The distance between the earth and the Sun is 1.5×10^{11} m. If the gravitational force between the two is 3.5×10^{22} N, what is the mass of the Sun? Use $G = 6.7 \times 10^{-11}$ N·m²kg⁻².**

Solution : Data : $m_1 = 6 \times 10^{24}$ kg, $r = 1.5 \times 10^{11}$ m, $F = 3.5 \times 10^{22}$ N, $G = 6.7 \times 10^{-11}$ N·m²kg^{-2}, $m_2 = ?$

$F = \frac{Gm_1m_2}{r^2}$

$\therefore m_2 = \frac{Fr^2}{Gm_1} = \frac{3.5 \times 10^{22} \text{ N} \times (1.5 \times 10^{11} \text{ m})^2}{6.7 \times 10^{-11} \text{ N·m}^2\text{kg}^{-2} \times 6 \times 10^{24} \text{ kg}}$

$= \frac{3.5 \times 1.5 \times 1.5 \times 10^{44}}{6.7 \times 6 \times 10^{13}}$ kg

$=$ **1.96×10^{30} kg (mass of the Sun).**

(10) Find the magnitude of the acceleration due to gravity at the surface of the earth. ($M = 6 \times 10^{24}$ kg, $R = 6400$ km)

Solution : Data : $M = 6 \times 10^{24}$ kg, $R = 6400$ km $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N·m² / kg², $g = ?$

$g = \frac{GM}{R^2}$

$= \frac{6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 6 \times 10^{24} \text{ kg}}{(6.4 \times 10^6 \text{ m})^2}$

$= \frac{66.7 \times 6}{(6.4)^2} \text{ m/s}^2 = 9.77 \text{ m/s}^2$

The magnitude of the acceleration due to gravity at the surface of the earth = 9.77 m / s².

***(11) The radius of planet A is half the radius of planet B. If the mass of A is M_A, what must be the mass of B so that the value of g on B is half that of its value on A?**

Solution : Data : $R_A = R_B/2$, $g_B = \frac{1}{2}g_A$, $M_B = ?$

$g = \frac{GM}{R^2}$ $\therefore g_A = \frac{GM_A}{R_A^2}$ and $g_B = \frac{GM_B}{R_B^2}$

$\therefore \frac{g_B}{g_A} = \left(\frac{M_B}{M_A}\right)\left(\frac{R_A}{R_B}\right)^2$

$\therefore \frac{1}{2} = \left(\frac{M_B}{M_A}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{4}\left(\frac{M_B}{M_A}\right)$

$\therefore \frac{M_B}{M_A} = \frac{4}{2} = 2$

$\therefore$ **$M_B = 2M_A$.**

*(12) **An object takes 5 s to reach the ground from a height of 5 m on a planet. What is the value of g on the planet?**

Solution : Data : $u = 0$ m/s, $s = 5$ m, $t = 5$ s, $g = ?$

$$\therefore s = \frac{1}{2}gt^2$$

$$\therefore 5 \text{ m} = \frac{1}{2}g \times (5 \text{ s})^2 = \frac{1}{2}g \times 5 \text{ s} \times 5 \text{ s}$$

$$\therefore g = \frac{2}{5} \text{ m/s}^2 = \mathbf{0.4 \text{ m/s}^2} \text{ (on the planet).}$$

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(13) **The mass of an imaginary planet is 3 times the mass of the earth. Its diameter is 25600 km and the earth's diameter is 12800 km. Find the value of the acceleration due to gravity at the surface of the planet.**

[g(earth) = 9.8 m / s^2]

Solution : Data : $\frac{M_2 \text{ (planet)}}{M_1 \text{ (earth)}} = 3,$

D_1 (earth) = 12800 km

$$\therefore R_1 \text{ (earth)} = \frac{12800 \text{ km}}{2} = 6400 \text{ km}$$

$$= 6.4 \times 10^6 \text{ m}$$

D_2 (planet) = 25600 km

$$\therefore R_2 \text{ (planet)} = \frac{25600 \text{ km}}{2} = 12800 \text{ km},$$

$$= 1.28 \times 10^7 \text{ m}$$

g_1 (earth) = 9.8 m / s^2, g_2 (planet) = ?

$$g = \frac{GM}{R^2} \quad \therefore g_1 = \frac{GM_1}{R_1^2}, \; g_2 = \frac{GM_2}{R_2^2}$$

$$\therefore \frac{g_2}{g_1} = \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right)^2$$

$$\therefore g_2 = g_1 \left(\frac{M_2}{M_1}\right)\left(\frac{R_1}{R_2}\right)^2$$

$$= 9.8 \text{ m/s}^2 \times 3 \times \left(\frac{6.4 \times 10^6 \text{ m}}{1.28 \times 10^7 \text{ m}}\right)^2$$

$$= \frac{9.8 \times 3}{4} \text{ m/s}^2 = 7.35 \text{ m/s}^2$$

The value of the acceleration due to gravity at the surface of the planet = 7.35 m/s^2.

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(14) **If the acceleration due to gravity on the surface of the earth is 9.8 m/s^2, what will be the acceleration due to gravity on the surface of a planet whose mass and radius both are two times the corresponding quantities for the earth?**

Solution : Data : $g_e = 9.8$ m/s^2, $M_p = 2M_e$, $R_p = 2R_e$, $g_p = ?$

Acceleration due to gravity, $g = \frac{GM}{R^2}$

$$\therefore g_e = \frac{GM_e}{R_e^2} \text{ and } g_p = \frac{GM_p}{R_p^2}$$

$$\therefore \frac{g_p}{g_e} = \left(\frac{M_p}{M_e}\right)\left(\frac{R_e}{R_p}\right)^2 = 2 \times \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\therefore g_p = \frac{g_e}{2} = \frac{9.8 \text{ m/s}^2}{2} = 4.9 \text{ m/s}^2$$

The acceleration due to gravity on the surface of the planet = 4.9 m/s^2.

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*(15) **A stone thrown vertically upwards with initial velocity u reaches a height h before coming down. Show that the time taken to go up is same as the time taken to come down.**

Solution :

We have,

$v = u + at$ ... (1)

and $s = ut + \frac{1}{2}at^2$ ... (2)

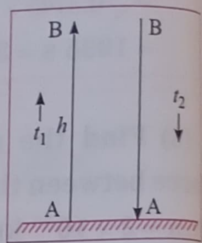

Fig. 1.9 : A → B, the stone moves upward; B → A, the stone moves downward

$$\therefore s = (v - at)t + \frac{1}{2}at^2$$

$$= vt - at^2 + \frac{1}{2}at^2$$

$\therefore s = vt - \frac{1}{2}at^2$ ... (3)

As the stone moves upward from A → B,

$s = AB = h$, $t = t_1$,

$a = -g$ (retardation),

$u = u$ and $v = 0$

$\therefore$ From Eq. (3), $h = 0 - \frac{1}{2}(-g)\,t_1^{\,2}$

$\therefore h = \frac{1}{2}gt_1^{\,2}$ ... (4)

As the stone moves downward from B → A,

$t = t_2$, $u = 0$, $s = h$ and $a = g$

$\therefore$ from Eq. (2), $h = \frac{1}{2}gt_2^{\,2}$ ... (5)

From Eqs. (4) and (5), $t_1^{\,2} = t_2^{\,2}$

$\therefore t_1 = t_2$ ($\because$ t_1 and t_2 are positive)

*(16) An object thrown vertically upwards reaches a height of 500 m. What was its initial velocity? How long will the object take to come back to the earth? Assume $g = 10\ m/s^2$.

Solution : Data : $h = 500$ m, $g = 10\ m/s^2$, $v = 0$ m/s, $u = ?$, t (for going up) + t (for coming down) = ?

As the object moves upward,

$v^2 = u^2 + 2as$

$= u^2 + 2(-g)h \quad (\because a = -g)$

Now, $v = 0$ m/s

$\therefore u^2 = 2gh = 2 \times 10\ m/s^2 \times 500$ m

$\therefore u^2 = (100 \times 100)\ m/s^2$

$\therefore u = 100$ m/s **(initial velocity of the body)**

Also, $v = u + at = u - gt$

For $v = 0$ m/s, $u = gt$

$\therefore 100\ m/s = 10\ m/s^2 \times t \quad \therefore t$ (for going up) = 10 s

Now, t (for coming down) = t (for going up) = 10 s

$\therefore t$ (for going up) + t (for coming down)

= 10 s + 10 s = 20 s

It will take 20 s for the object to come back to the earth.

***(17) A ball falls off a table and reaches the ground in 1 s. Assuming $g = 10\ m/s^2$, calculate its speed on reaching the ground and height of the table.**

Solution : Data : $t = 1 s$, $g = 10\ m/s^2$, $u = 0$ m/s, $s = ?$, $v = ?$

(i) $s = ut + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$ for $u = 0$ m/s

$\therefore s = \frac{1}{2} \times 10\ m/s^2 \times (1\ s)^2 = 5$ m

∴ The height of the table = 5 m.

(ii) $v = u + at = u + gt$

$= 0\ m/s + 10\ m/s^2 \times 1$ s

= 10 m/s

∴ The velocity of the ball on reaching the ground = 10 m/s.

(18) A body is released from the top of a building of height 19.6 m. Find the velocity with which the body hits the ground.

Solution : Data : $h = 19.6$ m, $u = 0$ m/s, $g = 9.8\ m/s^2$, $s = 19.6$ m, $v = ?$

$v^2 = u^2 + 2\,gs$

$= 2\,gs \quad ...$ (as $u = 0$ m/s)

$= 2 \times 9.8\ m/s^2 \times 19.6$ m

$= (19.6\ m/s)^2$

$\therefore v = 19.6$ m/s (downward velocity)

The velocity with which the body hits the ground = 19.6 m/s (downward).

(19) A stone on a bridge on a river falls into the river. If it takes 3 seconds to reach the surface of water, find (i) the velocity of the stone at the instant it touches the surface of water (ii) the height of the bridge from the surface of water.

Solution : Data : $u = 0$ m/s, $t = 3$ s, $g = 9.8\ m/s^2$, $v = ?$, $h = ?$

(i) $v = u + gt = 0\ m/s + 9.8\ m/s^2 \times 3$ s

= 29.4 m/s

The velocity of the stone at the instant it touches the surface of water = 29.4 m/s

(ii) $s = ut + \frac{1}{2}gt^2$

$= 0\ m/s \times 3\ s + \frac{1}{2}\ (9.8\ m/s^2)\ (3\ s)^2$

$= 4.9 \times 9\ m = 44.1$ m

∴ The height of the bridge from the surface of water = 44.1 m.

(20) A stone is dropped from rest from the top of a building 44.1 m high. It takes 3 s to reach the ground. Use this information to calculate the value of g.

Solution : Data : $u = 0$ m/s, $h = 44.1$ m

$\therefore s = 44.1$ m, $t = 3$ s, $g = ?$

$s = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad (\because u = 0\ m/s)$

$\therefore a = \frac{2s}{t^2} = \frac{2 \times (44.1\ m)}{(3\ s)^2}$

$= \frac{88.2}{9}\ m/s^2 = 9.8\ m/s^2$. It is the acceleration due gravity.

The value of $g = 9.8\ m/s^2$.

(21) **If the weight of a body on the surface of the moon is 100 N, what is its mass?**

($g = 1.63\ m/s^2$)

Solution : Data : $W = 100$ N, $g = 1.63$ m/s^2, $m = ?$

$W = mg$

$\therefore m = \frac{W}{g} = \frac{100\ N}{1.63\ m/s^2} = 61.35$ kg

The mass of the body = 61.35 kg.

(22) **A 100 kg bag of wheat is placed on a plank of wood. What is the weight of the bag and what is the reaction force exerted by the plank?**

Solution : Data : $m = 100$ kg, $g = 9.8$ m/s^2, $W = ?$, reaction force = ?

Magnitude of the weight,

$W = mg = 100$ kg $\times$ 9.8 m/s^2

$= 980$ N

The weight of the bag = 980 N acting downward.

The reaction force exerted by the plank on the bag = 980 N acting upward.

*(23) **The mass and weight of an object on earth are 5 kg and 49 N respectively. What will be their values on the moon? Assume that the acceleration due to gravity on the moon is 1/6th of that on the earth.**

Solution : Data : $m = 5$ kg, $W = 49$ N,

$g_M = \frac{g_E}{6}$, m (on the moon) = ?, W(on the moon) = ?

(i) The mass of the object on the moon = the mass of the object on the earth = 5 kg

(ii) $W = mg$

$\therefore \frac{W_M}{W_E} = \frac{mg_M}{mg_E} = \frac{g_M}{g_E} = \frac{1}{6}$

$\therefore W_M = \frac{W_E}{6} = \frac{49\ N}{6} = 8.167$ N (weight of the object on the moon).

(24) **Find the gravitational potential energy of a body of mass 10 kg when it is on the earth's surface. [M(earth) $= 6 \times 10^{24}$ kg, R(earth) $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2]**

Solution : Data : $m = 10$ kg, $M = 6 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

The gravitational potential energy of the body

$= -\frac{GMm}{R}$

$= -\frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 6 \times 10^{24}\ kg \times 10\ kg}{6.4 \times 10^6\ m}$

$= -\frac{6.67 \times 6}{6.4} \times 10^8\ J = -6.253 \times 10^8$ J.

(25) **If the body in Ex. (24) performs uniform circular motion around the earth at a height of 3600 km from the earth's surface, what will be its gravitational potential energy?**

Solution : Here, $h = 3600$ km $= 3.6 \times 10^6$ m

$\therefore$ The gravitational potential energy of the body

$= -\frac{GMm}{R+h}$

$= -\frac{6.67 \times 10^{-11}\ N \cdot m^2/kg^2 \times 6 \times 10^{24}\ kg \times 10\ kg}{(6.4 \times 10^6 + 3.6 \times 10^6)\ m}$

$= -\frac{6.67 \times 6 \times 10^{14}}{10 \times 10^6}\ J = -4.002 \times 10^8$ J.

(26) **A body of mass 20 kg is at rest on the earth's surface. (i) Find its gravitational potential energy. (ii) Find the kinetic energy to be provided to the body to make it free from the gravitational influence of the earth.**

($g = 9.8$ m/s^2, $R = 6400$ km)

Solution : Data : $m = 20$ kg, $g = 9.8$ m/s^2,

$R = 6400$ km $= 6.4 \times 10^6$ m

(i) The gravitational potential energy of the body =

$-\frac{GMm}{R} = -mgR \quad \ldots \left(\because g = \frac{GM}{R^2}\right)$

$= -20$ kg $\times$ 9.8 m/s^2 $\times$ 6.4×10^6 m

$= -1.2544 \times 10^9$ J.

(ii) To make the body free from the gravitational influence of the earth, it should be provided kinetic energy equal to 1.2544×10^9 J.

(27) If the body in Ex. (26) is moving at 100 m/s on the earth's surface, what will be its (i) kinetic energy (ii) total energy?

Solution : Data : $m = 20$ kg, $v = 100$ m/s

(i) The kinetic energy of the body

$$= \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \text{ kg} \times (100 \text{ m/s})^2 = 10^5 \text{ J.}$$

(ii) The total energy of the body = kinetic energy + potential energy = 10^5 J + (-1.2544×10^9 J)

$= (1 - 12544) \times 10^5 \text{ J} = -12543 \times 10^5$ J

$= -1.2543 \times 10^9$ J.

(28) A satellite of mass 100 kg performs uniform circular motion around the earth at a height of 6400 km from the earth's surface. Find its gravitational potential energy.

[$g = 9.8$ m/s^2, $R = 6400$ km]

Solution : Data : $m = 100$ kg, $g = 9.8$ m/s^2,

$R = 6400$ km $= 6.4 \times 10^6$ m, $h = 6.4 \times 10^6$ m

The gravitational potential energy of the satellite

$$= -\frac{GMm}{R+h} = -\frac{mgR^2}{R+h} \quad \dots \left(\because g = \frac{GM}{R^2}\right)$$

$$= -\frac{100 \text{ kg} \times 9.8 \text{ m/s}^2 \times (6.4 \times 10^6 \text{ m})^2}{(6.4 \times 10^6 + 6.4 \times 10^6) \text{ m}}$$

$$= -\frac{9.8 \times 6.4 \times 6.4 \times 10^{14}}{2 \times 6.4 \times 10^6} \text{ J} = -9.8 \times 3.2 \times 10^8 \text{ J}$$

$= -3.136 \times 10^9$ J.

(29) Find the escape velocity of a body from the earth. [M(earth) $= 6 \times 10^{24}$ kg, E(earth) $= 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2]

Solution : Data : $M = 6 \times 10^{24}$ kg, $R = 6.4 \times 10^6$ m, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

The escape velocity of a body from the earth,

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 6 \times 10^{24} \text{ kg}}{6.4 \times 10^6 \text{ m}}}$$

$$= \sqrt{\frac{12 \times 6.67 \times 10^8}{64}}$$

$= 1.118 \times 10^4$ m/s $= 11.18$ km/s.

(30) Find the escape velocity of a body from the earth. [R(earth) $= 6.4 \times 10^6$ m, ρ(earth) $= 5.52 \times 10^3$ kg/m^3, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2]

Solution : Data : $R = 6.4 \times 10^6$ m,

$\rho = 5.52 \times 10^3$ kg/m^3, $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

The escape velocity of a body from the earth,

$$v_{esc} = 2R\sqrt{\frac{2}{3}G\pi\rho}$$

$$= 2 \times 6.4 \times 10^6 \text{ m} \times \sqrt{\frac{2}{3} \times 6.67 \times 10^{-11} \text{ N·m}^2/\text{kg}^2 \times 3.142 \times 5.52 \times 10^3 \text{ kg/m}^3}$$

$$= 1.28 \times 10^3 \times \sqrt{\frac{13.34 \times 3.142 \times 5.52}{3}}$$

$= 11.24 \times 10^3$ m/s

$= 11.24$ km/s.

(31) Calculate the escape velocity of a body from the moon. [g (moon) $= 1.67$ m/s^2, R(moon) $= 1.74 \times 10^6$ m]

Solution : Data : $g = 1.67$ m/s^2, $R = 1.74 \times 10^6$ m

The escape velocity of a body from the moon,

$$v_{esc} = \sqrt{2gR}$$

$$= \sqrt{2 \times 1.67 \text{ m/s}^2 \times 1.74 \times 10^6 \text{ m}}$$

$= 2.411 \times 10^3$ m/s $= 2.411$ km/s.

(32) The mass of (an imaginary) planet is four times that of the earth and its radius is double the radius of the earth. The escape velocity of a body from the earth is 11.2×10^3 m/s. Find the escape velocity of a body from the planet.

Solution : Data : $M_2 = 4M_1$, $R_2 = 2R_1$,

$v_{1esc} = 11.2 \times 10^3$ m/s

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$\therefore \frac{v_{2esc}}{v_{1esc}} = \sqrt{\frac{M_2}{M_1} \times \frac{R_1}{R_2}} = \sqrt{4 \times \frac{1}{2}} = \sqrt{2}$$

$\therefore v_{2esc} = \sqrt{2}\, v_{1esc} = 1.414 \times 11.2 \times 10^3$ m/s

$= 15.84 \times 10^3$ m/s $= 15.84$ km/s

This is the escape velocity of a body from the pla

If m is the mass of the planet, the centripetal force exerted on the planet by the Sun (= gravitational force), $F = \frac{mv^2}{r}$

$$\therefore F = \frac{m(2\pi r/T)^2}{r} = \frac{4\pi^2 mr^2}{T^2 r} = \frac{4\pi^2 mr}{T^2}$$

According to Kepler's third law,
$T^2 = Kr^3$

$$\therefore F = \frac{4\pi^2 mr}{Kr^3} = \frac{4\pi^2 m}{K}\left(\frac{1}{r^2}\right)$$

Thus, $F \propto \frac{1}{r^2}$.

• Use your brain power! **(Textbook page 4)**

Q. If the area ESF in figure 1.3 is equal to area ASB, what will you infer about EF?

Ans. The time taken by the planet to move from E to F equals the time taken by the planet to move from A to B.

(5) Explain the term gravitational force. What is gravitation?

Ans. There exists a force of attraction between any two particles of matter in the universe such that the force depends only on the masses of the particles and the separation between them. It is called the gravitational force and the mutual attraction is called gravitation.

***(6) Let the period of revolution of a planet at a distance R from a star be T. Prove that if it was at a distance of $2R$ from the star, its period of revolution will be $\sqrt{8}T$.**

Ans. $T = \frac{2\pi}{\sqrt{GM}} r^{3/2}$, where T = period of revolution of a planet around the Sun, M = mass of the Sun, G = gravitational constant and r = radius of the orbit assumed to be circular = distance of the planet from the Sun.

For $r = R$ and $T = T_1$,

$$T_1 = \frac{2\pi}{\sqrt{GM}} R^{3/2}$$

For $r = 2R$ and $T = T_2$,

$$T_2 = \frac{2\pi}{\sqrt{GM}} (2R)^{3/2} = \frac{2\pi}{\sqrt{GM}} R^{3/2} \times 2^{3/2}$$

$$\therefore T_2 = T_1\sqrt{8} = \sqrt{8}T.$$

(7) State Newton's universal law of gravitation. Express it in mathematical form.

Ans. Newton's universal law of gravitation: Every object in the Universe attracts every other object with a definite force. This force is directly proportional to the product of the masses of the two objects and inversely proportional to the square of the distance between them.

Mathematical form: Consider two objects of masses m_1 and m_2. We assume that the objects are very small spheres of uniform density and the distance r between their centres is very large compared to the radii of the spheres (Fig. 1.5).

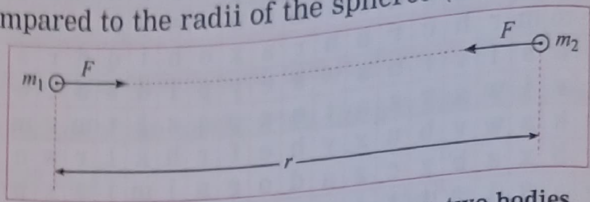

Fig. 1.5 : Gravitational force between two bodies

The magnitude (F) of the gravitational force of attraction between the objects is directly proportional to $m_1 m_2$ and inversely proportional to r^2

$$\therefore F \propto \frac{m_1 m_2}{r^2} \qquad \therefore F = G\frac{m_1 m_2}{r^2}$$

where G is the constant of proportionality, called the universal gravitational constant.

[Note : In the textbook, the word object/body is used. Newton's law of gravitation applies to particles.]

(8) (i) Why is the constant of gravitation called a universal constant?

(ii) Newton's law of gravitation is called the universal law of gravitation. Why?

Ans. (i) The value of the constant of gravitation, does not change with the nature, mass or the size of the material particles. It does not vary with the distance between the two particles. It is also independent of the nature of the medium between the two particles. Hence, it is called a universal constant.

(ii) As the law of gravitation given by Newton is applicable throughout the universe and to all particles, it is called universal law.

[Note : The centre of mass of an object is the point inside or outside the object at which the total mass of the object can be assumed to be concentrated to study the effect of an applied force.]

Q. 7 Some of the important terms in the chapter Gravitation are given in the following box. Find them :

f	r	l	d	o	p	r	s	a	x	l	k	t	s	t	j	u	p	e	n	o
c	b	m	a	c	b	e	c	f	d	g	e	s	t	u	i	v	e	m	s	l
e	c	b	u	a	v	w	e	b	w	f	v	u	p	r	o	p	r	n	r	k
n	n	z	d	l	e	h	m	f	n	g	o	v	y	w	h	k	i	g	j	q
t	o	c	i	f	l	w	i	x	b	a	d	x	q	s	t	n	o	s	i	p
r	p	m	i	j	g	n	z	o	y	p	i	y	z	k	l	l	d	h	h	m
i	s	b	h	e	b	h	e	g	a	b	y	z	a	m	f	j	i	g	f	g
p	t	h	g	f	a	i	f	x	z	c	t	d	b	u	j	m	c	n	i	o
e	s	c	a	p	e	v	e	l	o	c	i	t	y	v	e	l	t	p	e	a
t	u	o	l	k	j	n	f	e	k	o	e	s	q	w	p	o	i	o	d	j
a	p	o	d	p	e	m	l	d	j	l	f	q	u	z	f	n	m	p	c	q
l	q	n	e	q	d	g	d	c	i	m	g	n	v	w	y	r	e	q	k	b
f	r	m	m	r	h	c	r	b	h	r	a	x	o	h	l	q	d	r	s	a
o	s	w	s	i	v	t	u	s	w	s	c	y	p	i	d	s	o	t	u	l
r	t	v	j	w	x	y	z	a	t	q	z	p	q	z	j	t	m	v	m	z
c	u	k	a	w	v	b	u	x	y	b	e	f	r	h	k	l	r	w	n	o
e	y	b	x	a	b	z	c	z	a	d	o	g	n	i	m	j	s	x	p	y
g	r	a	v	i	t	a	t	i	o	n	a	l	c	o	n	s	t	a	n	t

Ans.

I	II	III
Mass	kg	Measure of inertia
Weight	N	Depends on height
Acceleration due to gravity	m/s^2	Zero at the centre of the earth
Gravitational constant	$N \cdot m^2/kg^2$	Same in the entire universe

Q. 5 Match the following :

Column A	Column B
(1) Escape velocity	(a) $\frac{-GMm}{(R+h)}$
(2) Gravitational acceleration	(b) $\sqrt{\frac{2GM}{R}}$
(3) Gravitational potential energy	(c) $\frac{Gm_1m_2}{r^2}$
(4) Gravitational force	(d) $\frac{GM}{r^2}$ $(r \geqslant R)$
	(e) $\frac{-GMm}{2(R+h)}$

Ans. (1) Escape velocity : $\sqrt{\frac{2GM}{R}}$

(2) Gravitational acceleration : $\frac{GM}{r^2}$ $(r \geqslant R)$

(3) Gravitational potential energy : $\frac{-GMm}{(R+h)}$

(4) Gravitational force : $\frac{Gm_1m_2}{r^2}$.

Q. 6 Answer the following questions in one sentence each :

(1) State the SI and CGS units of *G*.

Ans. The SI unit of G is $N \cdot m^2/kg^2$ and CGS unit is the $dyne \cdot cm^2/g^2$.

(2) State any one characteristic of gravitational force.

Ans. Gravitational force between two particles does not depend on the nature of the medium between them.

(3) Name the force that keeps the satellite in the orbit around the earth.

Ans. The gravitational force due to the earth keeps the satellite in the orbit around the earth.

(4) Name the force due to which the earth revolves around the Sun.

Ans. The earth revolves around the Sun due to the gravitational force of attraction exerted on it by the Sun.

(5) What is the acceleration due to gravity at a height *h* (= radius of the earth) from the surface of the earth? ($g = 9.8\ m/s^2$)

Ans. The acceleration due to gravity at a height h (= radius of the earth) from the surface of the earth is $2.45\ m/s^2$.

[**Explanation :** $g' = \frac{GM}{(R+h)^2} = \frac{GM}{4R^2} = \frac{g}{4}$

$= \frac{9.8}{4}\ m/s^2 = 2.45\ m/s^2$ for $h = R$]

(6) What is the relation between the SI unit of weight and the CGS unit of weight?

Ans. The relation between the SI unit of weight (the newton) and the CGS unit of weight (the dyne) is 1 newton $= 10^5$ dynes.

(7) Write the formula for the centripetal force acting on a body performing circular motion.

Ans. $F = \frac{mv^2}{r}$.

(8) Write the formula for the escape velocity of a body from the earth's surface.

Ans. $v_{esc} = \sqrt{\frac{2GM}{R}}$ or $v_{esc} = \sqrt{2gR}$.

(9) What is the value of the acceleration due to gravity at the centre of the earth?

Ans. Zero.

(10) What are the factors on which the maximum height attained by a body thrown upwards depends?

Ans. The initial velocity of the body, the acceleration due to gravity at that place, the buoyant force and frictional force due to air.

Q. 2 Select the correct option and write the completed statements :

(1) The gravitational force between two particles separated by a distance r varies as ....

(a) $\frac{1}{r}$ (b) r (c) r^2 (d) $\frac{1}{r^2}$

(2) In the usual notation, the acceleration due to gravity at a height h from the surface of the earth is ....

(a) $g = \frac{GM}{(R+h)}$ (b) $g = \frac{GM}{\sqrt{R+h}}$

(c) $g = \frac{GM}{(R+h)^2}$ (d) $g = GM(R+h)^2$

(3) The SI unit of the universal constant of gravitation is ....

(a) $N \cdot m^2/kg^2$ (b) $N \cdot kg^2/m^2$

(c) m/s^2 (d) $kg \cdot m/s^2$

(4) The escape velocity of a body from the earth's surface, v_{esc} = ...

(a) $\sqrt{\frac{GM}{R}}$ (b) $2\sqrt{\frac{GM}{R}}$

(c) $\sqrt{\frac{2GM}{R}}$ (d) $\sqrt{\frac{GM}{2R}}$

Ans.

(1) The gravitational force between two particles separated by a distance r varies as $\frac{1}{r^2}$.

(2) In the usual notation, the acceleration due to gravity at a height h from the surface of the earth is $g = \frac{GM}{(R+h)^2}$.

(3) The SI unit of the universal constant of gravitation is $N \cdot m^2/kg^2$.

(4) The escape velocity of a body from the earth's surface, $v_{esc} = \sqrt{\frac{2GM}{R}}$.

Q. 3 State whether the following statements are *True* or *False* : (If a statement is false, correct it and rewrite it.)

(1) If the separation between two particles is doubled, the gravitational force between the particles becomes half the initial force.

(2) The CGS unit of the universal constant of gravitation is the $dyne \cdot cm^2/gram^2$.

(3) At the centre of the earth, the value of the acceleration due to gravity becomes zero.

(4) The weight of a body is minimum at the poles.

(5) Mass is a vector quantity.

(6) Weight is a vector quantity.

(7) g has maximum value at the equator.

(8) Outside the earth, g varies as $1/(R+h)^2$.

(9) The value of G changes from place to place.

(10) The value of g increases with altitude.

(11) The escape velocity of a body does not depend on the mass of the body.

(12) The mass of a body is the amount of matter present in it.

Ans.

(1) **False.** (If the separation between two particles is doubled, the gravitational force between the particles becomes $\frac{1}{4}$ times the initial force.)

(2) **True.** (3) **True.**

(4) **False.** (The weight of a body is maximum at the poles.)

(5) **False.** (Mass is a scalar quantity.)

(6) **True.**

(7) **False.** (g has maximum value at the poles.)

(8) **True.**

(9) **False.** (The value of G is the same throughout the universe.)

(10) **False.** (The value of g decreases with altitude.)

(11) **True** (12) **True.**

***Q. 4 Study the entries in the following table and rewrite them putting the connected items in a single row :**

I	II	III
Mass	m/s^2	Zero at the centre of the earth
Weight	kg	Measure of inertia
Acceleration due to gravity	$N \cdot m^2/kg^2$	Same in the entire universe
Gravitational constant	N	Depends on height

Q. 1 Fill in the blanks with appropriate words and write the completed sentences :

(1) The ratio $g_{(earth)}/g_{(moon)}$ is equal to ....

(2) The value of the acceleration due to gravity .... as we move from the equator to a pole.

(3) If the earth shrinks to half of its radius, its mass remaining the same, the weight of an object on the earth will become .... times.

(4) The SI unit of weight is the ....

(5) The CGS unit of weight is the ....

(6) The weight of a body is .... at the poles.

(7) Outside the earth, the weight of a body varies as ....

(8) Due to the .... force, the earth attracts all objects towards it.

(9) The acceleration due to gravity does not depend on the .... of the body.

(10) According to Kepler's first law, the orbit of a planet is .... with the Sun at one of the ....

(11) According to Kepler's second law, the line joining the planet and the Sun .... in equal intervals of time.

(12) According to Kepler's third law $T^2 \propto r^n$, where $n =$ ....

Ans.

(1) The ratio $g_{(earth)}/g_{(moon)}$ is equal to 6 (approximately).

(2) The value of the acceleration due to gravity increases as we move from the equator to a pole.

(3) If the earth shrinks to half of its radius, its mass remaining the same, the weight of an object on the earth will become four times.

(4) The SI unit of weight is the newton.

(5) The CGS unit of weight is the dyne.

(6) The weight of a body is maximum at the poles.

(7) Outside the earth, the weight of a body varies as $1/(R+h)^2$.

(8) Due to the gravitational force, the earth attracts all objects towards it.

(9) The acceleration due to gravity does not depend on the mass of the body.

(10) According to Kepler's first law, the orbit of a planet is an ellipse, with the Sun at one of the foci.

(11) According to Kepler's second law, the line joining the planet and the Sun sweeps equal areas in equal intervals of time.

(12) According to Kepler's third law $T^2 \propto r^n$, where $n = 3$.

[**Note :** (1) The questions marked with an asterisk (*) are textual questions. (2) HOTS (Higher Order Thinking Skill Questions)]